Effects of Fertilizer Programs on the ECONOMIC CHOICE OF CROPS in Selected Areas of Illinois

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Effects of Fertilizer Programs on the Economic Choice of Crops in Selected Areas of Illinois

M. F. JORDAN and C. B. BAKER[1]

BETWEEN 1950 AND 1958, SOYBEAN ACREAGE INCREASED BY MORE than 30 percent in the cash-grain area of east-central Illinois (State Economic Area 6b), and by more than 10 percent in south-central Illinois (State Economic Area 8). (See Table 1.) Acreage in wheat increased by an even larger percentage in both areas, while decreases occurred in oats, hay, and corn. In these same years, there were changes in the price of livestock and crops, as well as in the technology of production. This bulletin reports the result of research in these areas to determine the effects on crop selection of changes in fertilizer programs.

An outline of the essential economics of crop selection provides the basis for interpreting the results.

ECONOMICS OF CROP SELECTION

A farmer's selection of crops depends mainly on two factors: the natural and technical resources available for cultivation, and the financial returns from their sale or use. The farmer's basic economics of choice may be conveniently schematized (see Fig. 2). From a given input of land, labor, and fixed capital applied to produce crop A, a maximum Oa might be produced. By shifting all resources to crop B, Ob could be produced. The curve connecting these two points represents combinations of A and B that can be produced with fixed resources. The fact that it rises as it leaves the vertical axis indicates that, by producing crop B over a small range of production, crop A is so benefited that its production actually increases. Such a relation is "complementary": B complements A over this range. Thereafter, producing more of B requires a reduction in A. Thus B is said to compete with A over a wide range of production. Since crop production is subject to the "law of diminishing returns" with respect to most

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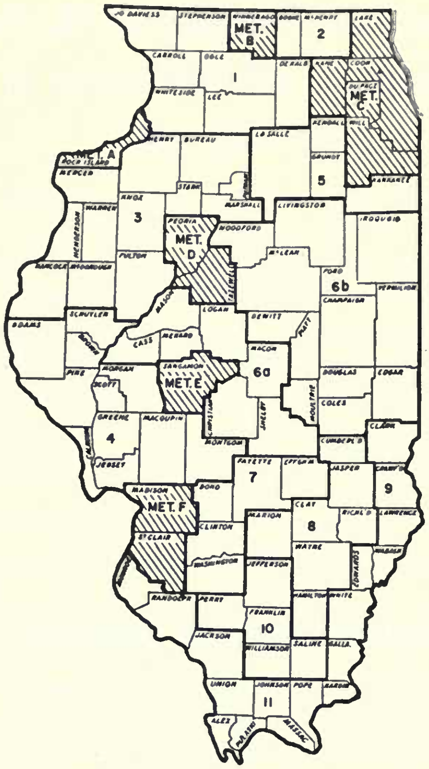
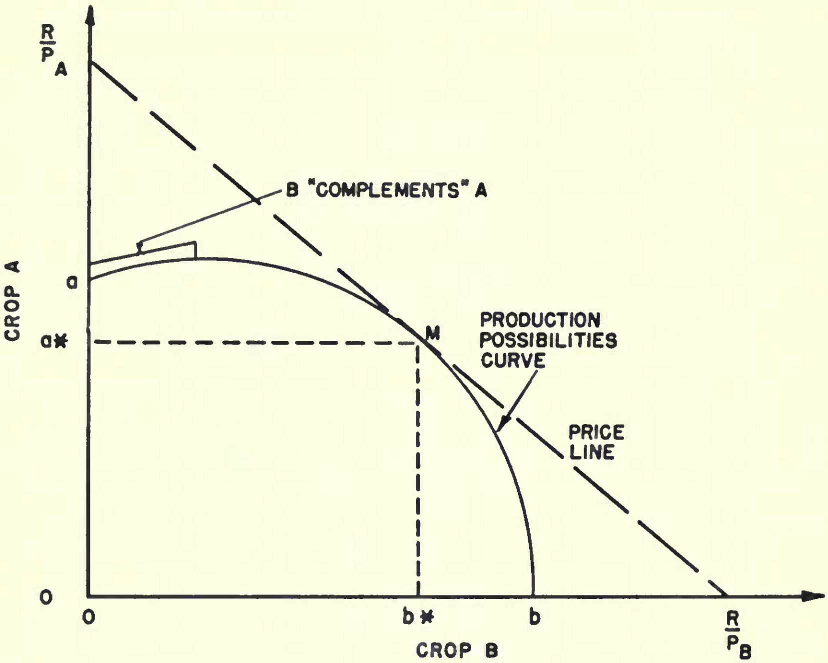


Illinois state economic areas. Metropolitan areas shown as shaded areas. (Fig. 1)

Table 1. — Relative Changes in Acreages of Principal Crops in Illinois Economic Areas 6b and 8, 1950-1958[a]

Crop	Area 6b 1958	Change 1950–58	Area 8 1958	Change 1950–58
	(acres)	(percentage)	(acres)	(percentage)
Corn	1,814,600	−1.90	314,500	−8.55
Soybeans	1,383,500	+32.49	392,200	+10.39
Wheat	255,600	+82.05	164,500	+153.85
Oats	573,000	−38.18	16,500	−79.71
Hay	306,500	−14.12	134,700	−24.62

[a] Illinois Cooperative Crop Reporting Service, *Ill. Agr. Stat.* Annual Summary Bulletins 51-1 and 59-1.



A schematization of the economics of choice. (Fig. 2)

resources, the usual relation expected between crops is one of competition. The curve described above represents the response of two crops to an input of a fixed quantity of resources that can be allocated between the two crops.

Revenue generated by producing crop A, crop B, or combinations of the two, is defined as R. To earn R (a given amount of revenue) by producing A alone would require producing A in an amount equal to R divided by the price of A (R/P_A). Comparably, R/P_B would be required of crop B to produce the same revenue (R). R cannot, in this example, be earned from either crop alone. But the combination of A (at Oa*) and B (at Ob*) does yield a revenue equal to R. In fact, it is the only combination that *does* yield so large an income.

All points on the line connecting R/P_B and R/P_A represent combinations of crops B and A that earn the same returns. The slope of the line represents the rate at which one crop substitutes for the other in terms of income. All points on the curve connecting Oa and Ob represent combinations of crops A and B that can be produced with the available resources. The particular combination of Oa* and Ob* of crops A and B, respectively, maximizes returns above the cost of these fixed resources.

Some resources are not allocable between crops. For example, crop seed, some harvesting equipment, etc., are specific to one crop or the other. To take the cost of these specific resources into account, the crop price can be reduced by their cost per unit of crop production. The result is a revenue per unit of output, above the cost of resources specific to the crop. The slope of the revenue line in Fig. 2 will be changed if the cost of such resources differs between crops. Hence, the combination of crops that maximizes returns also would be different: the optimum combination would shift toward the crop in which the specific costs are relatively less.

Each crop has certain specific nutrient requirements. Fertilizer applications also change the productivity of fixed resources. Therefore, the application of fertilizer affects the slope of both the revenue line (adjusted to account for the cost of specific resources) and the production possibility curve as well.

Differences between areas in the shift of production possibility curves may have significant implications. The advantage of farmers on good soils may increase or decrease relative to farmers on poor soils depending on production opportunities changed by fertilizer programs in each area. Consequences of these changes in the competitive positions of farmers in relatively large geographic areas may have important influence on regional specialization.

Linear programming models were used to estimate the effect on crop selection of changes that have occurred in fertilizer use. The models were also used to specify cropping alternatives in each area in 1954 and 1958, together with resources available to farmers typical of each area in these years. They then were used to select crop combinations which would maximize income in 1954 and 1958 when the cost of specific resources are subtracted from the price of crops. Then the prices of crops were varied and new estimates were made. From a series of such estimates it was possible to infer changes in characteristics of the production possibility relations among crops. Since estimates were desired that could be ascribed to changes in the fertilizer programs, an attempt was made to account for the effects of changes in nonfertilizer inputs over time. The primary changes were weather and mechanization. Before developing the analytic models in greater detail, we shall thus want to describe the areas, the data used, and the method of accounting for nonfertilizer sources of change in crop yields.

PRODUCTION CHARACTERISTICS OF THE AREAS

The east-central and south-central areas of Illinois which are under consideration in this study represent two types of farming areas that differ as to soil productivity, soil management, and cropping programs. These areas are shown on the map in Fig. 3. The east-central area has been for some time a highly productive cash-grain area, with a large proportion of its crop produced for market. The south-central area is a less productive farming area in which the agriculture has been classed as mixed or general farming. More recently, however, it has been shifting to a cash-grain area.[1]

East-central Illinois

Climate. Annual precipitation in east-central Illinois averages approximately 36 inches, with 55 to 60 percent occurring in the 170- to 180-day growing season.[2] Annual precipitation decreases from south to north in Illinois, though between the east-central and south-central areas the difference is mainly in greater winter precipitation in the south.

Mean July temperatures average 75.3° F., and mean January temperatures 26.8°.[3] At Urbana, in Champaign County, the average date of the last killing frost in the spring is April 21, and the average date of the first frost is October 19.[4]

Relative soil temperature also influences the crops that can be produced successfully in an area. Soil temperature, as well as air temperature, is an important factor in determining rate of plant growth and biological activity in the soil. Soil temperature may vary according to moisture conditions, color, compactness of the soil, the angle of exposure of the soil surface to sun rays, and vegetative or surface cover. The dark soils of central Illinois are more adequately drained and are warmer than those light-colored poorly drained soils of the claypan area. Thus, soil temperature, as well as air temperature and precipitation, may be said to have considerable influence on the cropping alternatives.

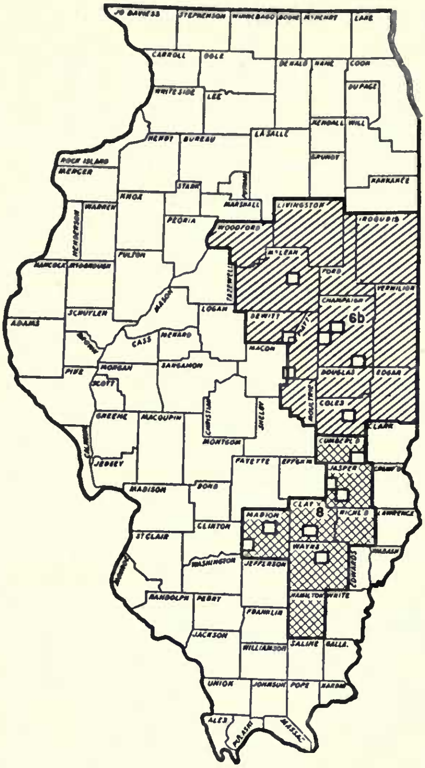
Soils in the east-central area are dark-colored prairie soils, underlain with medium-textured, moderately permeable subsoils. In general these

[1] See Table 2. On many southern Illinois farms the portion of the gross sales attributable to cash crop has increased at a rate greater than that of the gross sales attributable to livestock.

[2] Page, J. L., *Climate of Illinois,* Ill. Agr. Exp. Sta. Bul. 532, 1949, p. 138.

[3] *Ibid.,* p. 156, average at Urbana in Champaign County, 1889 to 1946.

[4] *Ibid.,* p. 128.



Location of economic areas and townships in the present survey. (Fig. 3)

soils developed from silty wind-blown loess.[1] The topography is level or gently sloping. The soils on the steeper slopes were developed from permeable glacial till rather than from loess. The soils of this east-central area are among the most productive in Illinois, and the major soil problems are mainly those of drainage and the proper maintenance of good physical condition. Many of these soils have tile drainage systems which function well in normal years. The drouth resistance of these soils is very good since conditions favor extensive root development.

Markets. Producers of grain, the main product of east-central Illinois agriculture, enjoy several market alternatives. Grain may be surrendered on Commodity Credit Corporation loan and purchase

[1] Wascher, H. L., et al. *Illinois Soil Type Description,* Ill. Agr. Exp. Sta. AG-1433, 1950, p. 175.

agreements, sold to other farmers, small feed mills, and truckers, or handled by country elevators. Most of the grain produced finds it way through the country elevators to terminal markets, and interior processors.[1]

Receipts in 1954-55 suggest that St. Louis and Peoria are the leading markets for east-central Illinois corn. St. Louis was the most important wheat market, handling nearly 50 percent of the production. Soybeans are processed at plants near production points. Decatur and other central Illinois processing points handle most of the soybeans. Oats from the area go to Chicago and St. Louis in about equal amounts with a considerable portion going by truck to Indiana and southern states.

Grain prices at St. Louis and Chicago are closely related, especially those for corn and oats. Both markets draw corn and oats at the same freight rates from the east-central area surpluses.

South-central Illinois

Climate. Annual precipitation in the south-central Illinois area averages approximately 40 inches, 50 to 55 percent occurring in the 180- to 190-day growing season.[2] Precipitation distribution throughout the growing season is especially critical on the soils of the area, as the following section will reveal.

Mean July temperatures average 79.8°, and mean January temperatures average 32.3°.[3] At Flora, in Clay County, the average date of the last killing frost in the spring is April 17 and the average date of the first frost is October 19.[4]

Soils in the south-central area are medium to light-colored prairie soils with very fine textured, very thin loess on weathered drift. The silt loam surface and subsurface soil is approximately 18 to 20 inches in depth, underlain with a gray mottled, heavy silty clay loam subsoil. Slow water penetration of this tight subsoil makes tiling impractical and excess water must be removed by open ditches. The topography is very gently sloping with slopes of 0.5 to 1.5 percent.[5] Evaporation is quite rapid from these soils, and, because of the shallow root systems developed by plants as a result of the tight subsoil, there is considerable damage to the crops during periods of drouth. Crops on these soils will

[1] Schumaier, C. P., *Illinois Grain Production and Trade,* Ill. Agr. Exp. Sta. Bul. 637, 1959, p. 28.

[2] Page, J. L., *op. cit.,* p. 138.

[3] *Ibid.,* p. 159.

[4] *Ibid.,* p. 128.

[5] Wascher, H. L., et al., *op. cit.,* p. 44.

suffer severely from a deficiency of moisture after 10 or 15 rainless days.[1] At the other weather extreme, with inadequate drainage, water may stand on some land for several days after a heavy rainfall. Therefore, because of the critical soil moisture problem, timeliness of operations is extremely important for successful crop production. These soils are usually lacking in natural fertility, attributable partly to the water problem, intensity of weathering, and decline of organic matter.

Markets. The grain markets for south-central Illinois are similar, and in some cases identical, to those of the east-central area. Expansion of the cash-grain area southward is apparent in the data on south-central Illinois grain markets. The volume of grain sold in the area in 1949-53 increased 2½ times over that sold in 1939-1943.[2]

Country elevators handle most of the off-farm sales of grain, with disposition similar to that of grains in the east-central area. The St. Louis terminal market and shipments to southern points account for most of the movements of corn from the country elevators. Considerable corn moves southward from the area by truck. Most of the wheat handled by country elevators moves to St. Louis. Some soybeans move to St. Louis from elevators in the southern area, but most of the soybeans go to Decatur and other Illinois processors. Oat production in the area is limited largely to winter oats and little is sold off-farm. Most of the off-farm sales are to truckers supplying southern Indiana and certain southern states.

THE PRESENT SURVEY

Selection of samples

The primary data for this study were obtained in a field survey of the economic areas indicated in Fig. 3. Congressional townships dominated by single soil associations were selected from a state soil association map. Flanagan soils dominate townships selected in the east-central area; Cisne soils, in the south-central area. Sections were selected randomly from the townships. All farms were visited that had fields in the sampled sections.

The survey

The survey was conducted from July 22 to October 31, 1958, the majority of the interviews being made during August and September.

[1] Cross, A. J., and Willis, J. E., *Organization and Operation of Farms in the Claypan Area of Southern Illinois,* Ill. Agr. Exp. Sta. Bul. 579, 1954, p. 4.
[2] Schumaier, C. P., *op. cit.,* p. 25.

Table 2. — Selected Statistics From Field Schedules: Flanagan and Cisne Soil Areas

Item	Flanagan soil	Cisne soil
Average age of operator	46.9	45.9
Average number of years on present farm unit	17.8	14.7
Average number of years of farm experience	22.2	19.8
Average number of farm tracts operated	1.7	2.0
Average acreage of cropland owned	135	89.2
Average acreage of cropland share rented by renters	205.3	121.3
Average acreage of cropland operated	214.4	148.1
Average acreage of all land owned	144.5	109.3
Percentage of units all owned	31.7	69.2
Average acreage of all land operated	230.7	175.8
Average acreage of all land share rented	220.8	142.4
Share of all crops to landlord (mode)	½	⅓
Percentage of crop share agreements in modal class	83.3	72.6
Percentage of fertilizer share agreements in modal class	82.5	59.5
Percentage of rental tracts consisting of a single tract	60.8	59.3
Percentage of rental tracts consisting of 2 tracts	25.5	22.2
Percentage of farms with family labor equivalent to 1 full-time operator	58.3	66.7
Percentage of farms reporting hired labor	21.7	12.8
Percentage of all farms reporting 1 full-time hired hand	8.3	5.1
Average number of months of hired labor on farms reporting it	9.0	6.0
Percentage of farms reporting operator working off farm more than 100 days per year	8.3	15.4

One hundred and eighty two completed schedules were obtained in the two areas, of which 96 were on the Flanagan soils and 86 on Cisne soils. The schedule was designed to obtain resource inventories for the farm business and cropping history of all fields.

Some of the personal characteristics of farmers, their tenure situation, and labor supply are summarized in Table 2. The proportions of farms reporting livestock, and the mean numbers of livestock per reporting farm for 1958, were quite low in both areas. (See Table 3.)

A comparison of crop machinery inventories shows that machines found on farms in the east-central area were of larger machine-unit capacity than those in the southern area. For example, the average combine in the Flanagan area had a 9-foot cutter-bar, while the mean length cutter-bar in the Cisne area was 6 feet. This same general size relationship was found in corn-picker capacity as well as planter and cultivator widths. About 88 percent of the farms in the east-central area reported some four-row equipment, while in the southern area only 31 percent reported one or more pieces of four-row equipment.

Table 3.—Proportion of Farms Reporting Livestock and Mean Numbers, by Type of Livestock and Area, July 1, 1958. Summary of all Farms Surveyed

Type of livestock	East-central area		South-central area	
	Percent of farms reporting	Average number of livestock reported	Percent of farms reporting	Average number of livestock reported
Dairy cows	23.3	2.5	35.9	7.9
Beef cows	49.2	10.3	51.3	10.7
Mature swine	37.3	9.3	53.8	5.2
Sheep	13.6	15.5	2.5	20
Poultry flocks	1.6	400	7.6	566
Cattle on feed	27.2	46.6	10.3	13
Spring pigs	33.9	84	41.1	38
Fall pigs	27.2	82	25.7	35
Feeder pigs purchased	13.5	82	9.6	10
Feeder cattle purchased	18.9	59	6.4	15

Inventories included quite a number of older model machines, especially in the southern area. The existing machinery, however, would generally be considered adequate for the units farmed.

Modal farm units

Surveyed farms in each area were arrayed in terms of land and labor. In both areas, farms were most numerous in the "141 through 180 acres" size class. The modal or typical farm class in each area also had one man-year of labor.

The typical unit on the Flanagan soils is a tenant-operated, single tract unit, of 172 acres under 50-50 crop-share agreement. Approximately five acres are in buildings and lots, seven acres are in permanent pasture, and the rest in crops. Livestock consists of five animal-units of forage-consuming livestock and three sows (six animal-units in all).

The typical unit on the Cisne soils is a single tract of 160 acres, owner-operated, with 136 acres of cropland. Twelve man-months of labor are available annually. Livestock consists of nine animal-units of forage-consuming livestock and two sows. The modal units were modal with respect to land area and labor supply. The specific farms comprising the modal classes were examined to determine the other characteristics noted here.

ANALYTIC MODEL

The typical farm for the east-central area is slightly larger (172 acres) than the modal for the southern area (160 acres). There is more nontilled land in the southern area, and also more livestock. The net result, shown in Table 4, is that 152 acres are available for competitive cropping in the east-central area, compared with 123 in the southern area.

Table 4. — Land Resources on Farms Modal for East-Central and South-Central Areas of Illinois (acres), 1958

	Area	
	East-central	South-central
Modal total acreage	172	160
Nontilled acreage	12	24
Cropland for livestock feed production	8[a]	13[a]
Cropland available for cropping	152	123

[a] Total corn equivalent feed requirement ÷ 1958 corn yield per acre in C-Sb rotation = cropland required for livestock feed production. See appendix Table 1 for livestock feed requirements.

For November through February, farms in each area are assumed to have a surplus of labor. The rest of the year is divided into labor periods as shown in Table 5. The total supply of 2,080 man hours is assumed to be similarly distributed over the labor periods for farms in both areas. However, the larger livestock requirement in the southern

Table 5. — Labor Available on Farms Modal for East-Central (Flanagan) and South-Central (Cisne) Areas of Illinois, by Labor Period (man-hours), 1958

Labor period	Total		Required for livestock and overhead		Available for competitive cropping	
	East-central	South-central	East-central	South-central	East-central	South-central
March	260	260	99	106	161	154
April–June	780	780	261	268	519	512
July–Sept.	780	780	299	313	481	467
October	260	260	92	99	168	161
Total	2,080	2,080	751	786	1,329	1,294

area leaves a smaller balance available for competitive cropping. The net labor supplies are given in the last two columns of Table 5.

Only a few farms in the modal classes reported hay harvested from clover. Instead the crop was used as green manure, with effects reflected in crop yields and fertilizer applications. A number of farms reported clipping weeds in clover. Hence, it was assumed that no hay was harvested and that weeds were clipped in the clover. In the model no change was allowed in the amount of crop carryover. Therefore, all crops produced were assumed sold. As noted above, the price was varied for each crop to observe differences in solutions generated by each of the numerous price relationships among crops.

Thus the alternatives available for land and labor resources consist of crop rotations found to exist in each area. These are arrayed in Table 6.

Fertilizer application rates found in the survey are reported in

Table 6. — Crop Rotations Found on Farms Surveyed in the East-Central and South-Central Cash-Grain Areas of Illinois, 1958

Crop rotation[a]	Area	
	East-central	South-central
Corn-soybeans-wheat-clover	x	x
Corn-soybeans-oats-clover	x	
Corn-soybeans-wheat (clover)	x	x
Corn-soybeans-oats (clover)	x	
Corn-soybeans	x	x
Corn-soybeans-oats	x	
Corn-soybeans-wheat	x	x
Corn-oats-clover	x	
Corn-corn-soybeans-oats-clover	x	
Corn-corn-soybeans-wheat (clover)	x	x
Corn-corn-soybeans-oats (clover)	x	
Corn-corn-soybeans	x	
Corn-corn-oats-clover	x	
Corn-corn-corn	x	x
Corn-soybeans-corn-oats	x	
Corn-soybeans-soybeans-wheat (clover)		x
Corn-soybeans-soybeans-wheat-clover		x
Wheat-soybeans		x
Corn-soybeans-redtop-redtop-redtop		x
Corn-redtop-redtop-redtop-soybeans		x
Soybeans-wheat-redtop-redtop-redtop-redtop		x
Soybeans-soybeans-soybeans		x

[a] Corn appears in all east-central rotations, and soybeans in all but three. In the south-central area, soybeans appear in all but one rotation. No oats were found in the south-central area; no redtop in the east-central area. Wheat is more important in the south-central area rotations.

Table 7. — Variable Costs per Rotation-Acre, by Crop and Fertilizer Nutrient, Flanagan Soils, 1954 and 1958

Crop rotation	Nonfertilizer costs						Fertilizer costs				Total, all variable costs
	Corn	Soybeans	Wheat	Oats	Clover	Total	Nitrogen	P_2O_5	K_2O	Total	
					1954						
P_1 C-Sb-W-Cl	$3.00	$2.62	$3.15	....	$2.17	$10.94	$ .82	$ .55	$ .27	$1.64	$12.58
P_2 C-Sb-O-Cl	2.96	2.43	....	$2.58	2.17	10.14	....	....	....	....	10.14
P_3 C-Sb-W(Cl)	5.20	3.21	3.56	....	2.33	14.30	2.17	.67	.34	3.18	17.48
P_4 C-Sb-O(Cl)	4.20	3.22	....	2.95	2.33	12.70	....	....	....	....	12.70
P_5 C-Sb	5.48	3.71	....	....	....	9.19	....	....	....	....	9.19
P_6 C-Sb-O	5.68	3.08	....	3.42	....	12.18	....	....	....	....	12.18
P_7 C-Sb-W	5.44	3.35	3.84	....	....	12.63	3.68	1.56	.78	6.02	18.65
P_8 C-O-Cl	5.28	....	....	2.14	2.90	10.32	2.80	....	....	....	13.12
P_9 C-C-Sb-O-Cl	5.05	1.70	....	1.32	1.75	9.82	....	....	....	....	9.82
P_{10} C-C-Sb-W(Cl)	6.27	2.48	2.30	....	1.75	12.80	1.63	1.22	.75	3.60	16.40
P_{11} C-C-Sb-O(Cl)	5.98	1.89	....	2.17	1.75	11.79	....	....	....	....	11.79
P_{12} C-C-Sb	9.04	3.24	....	....	....	12.28	1.20	.80	.40	2.40	14.68
P_{13} C-C-O-Cl	6.39	....	....	2.96	2.17	11.52	1.70	.80	.40	2.90	14.42
P_{14} C-C-C	9.99	....	....	....	....	9.99	....	....	....	....	9.99
P_{15} C-Sb-C-O	7.78	2.17	....	1.64	....	11.59	2.45	1.01	.51	3.97	15.56
					1958						
P_1 C-Sb-W-Cl	$3.21	$2.68	$3.68	....	$2.17	$11.74	$1.03	$ .81	$ .41	$2.25	$13.99
P_2 C-Sb-O-Cl	3.81	2.76	....	$3.17	2.17	11.91	1.88	.65	.43	2.96	14.87
P_3 C-Sb-W(Cl)	5.07	3.42	4.53	....	2.33	15.35	2.56	1.75	.85	5.16	20.51
P_4 C-Sb-O(Cl)	4.20	3.22	....	2.95	2.33	12.70	....	....	....	....	12.71
P_5 C-Sb	7.13	5.09	....	....	....	12.22	3.27	1.76	.47	5.50	17.72
P_6 C-Sb-O	6.68	3.95	....	4.12	....	14.75	2.11	.74	.37	3.22	17.97
P_7 C-Sb-W	5.78	3.43	4.68	....	....	13.89	3.64	2.12	1.04	6.80	20.69
P_8 C-O-Cl	4.85	....	....	3.28	2.90	11.03	1.28	1.24	.62	3.14	14.17
P_9 C-C-Sb-O-Cl	6.74	1.97	....	2.45	1.74	12.90	2.73	1.41	1.44	5.58	18.48
P_{10} C-C-Sb-W(Cl)	7.60	2.82	2.88	....	1.75	15.05	3.90	2.72	1.48	8.10	23.15
P_{11} C-C-Sb-O(Cl)	7.65	2.67	....	2.64	1.75	14.71	3.57	.61	.31	4.49	19.20
P_{12} C-C-Sb	11.34	3.62	....	....	....	14.96	6.08	2.50	.88	9.46	24.42
P_{13} C-C-O-Cl	7.38	....	....	2.28	2.17	11.83	2.19	....	.60	2.79	14.62
P_{14} C-C-C	11.53	....	....	....	....	11.53	2.30	1.22	.77	4.29	15.82
P_{15} C-Sb-C-O	7.97	2.17	....	1.37	....	11.51	2.50	.99	.25	3.74	15.25

Table 8. — Variable Costs per Rotation-Acre, by Crop and Fertilizer Nutrient, Cisne Soils, 1954 and 1958

Crop rotation	Nonfertilizer costs						Fertilizer costs				Total, all variable costs
	Corn	Soybeans	Wheat	Clover	Redtop	Total	Nitrogen	P_2O_5	K_2O	Total	
1954											
P_1 C-Sb-W-Cl	$1.70	$2.12	$1.66	$2.18	....	$ 7.66	$ .92	$ .98	$ .64	$2.54	$10.20
P_2 C-Sb-W(Cl)	3.26	2.81	2.07	2.33	....	10.47	2.36	3.12	3.49	8.97	19.44
P_3 C-Sb	3.47	3.76	....	....	....	7.23	.41	.94	.60	1.95	9.18
P_4 C-Sb-W	2.15	2.60	3.32	....	....	8.07	2.36	1.55	.77	4.68	12.75
P_5 C-Sb-Sb-W(Cl)	1.22	2.70	1.69	1.75	....	7.36	1.10	.48	.24	1.82	9.18
P_6 C-Sb-Sb-W-Cl	1.38	2.43	1.69	1.74	....	7.24	.34	.90	.45	1.69	8.93
P_7 W-Sb	....	2.19	3.96	....	....	6.15	....	2.40	....	2.40	8.55
P_8 C-C-Sb-W-Cl	3.17	1.22	1.90	1.74	....	8.03	.39	1.16	.58	2.13	10.16
P_9 C-C-Sb-W(Cl)	3.02	1.93	2.13	1.75	....	8.83	.51	1.46	.78	2.75	11.58
P_{10} C-Sb-Rt-Rt-Rt	1.17	1.50	....	....	2.16	4.83	....	....	....	....	4.83
P_{11} C-Rt-Rt-Rt-Sb	.63	1.24	....	....	2.16	4.03	....	....	....	....	4.03
P_{12} Sb-W-Rt-Rt-Rt-Rt	....	.84	.78	....	2.40	4.02	.19	.51	.26	.96	4.98
P_{13} C-C-C	6.26	....	....	....	....	6.26	.54	1.14	1.26	2.94	9.20
P_{14} Sb-Sb-Sb	....	7.02	....	....	....	7.02	....	....	....	....	7.02
1958											
P_1 C-Sb-W-Cl	$1.84	$2.14	$2.08	$2.18	....	$8.24	$1.23	$1.75	$ .65	$3.63	$11.87
P_2 C-Sb-W(Cl)	2.35	2.53	2.56	2.33	....	9.77	1.59	2.14	1.97	5.70	15.47
P_3 C-Sb	3.58	3.83	....	....	....	7.41	.56	.97	.79	2.32	9.73
P_4 C-Sb-W	2.35	2.61	3.10	....	....	8.06	2.40	1.71	1.15	5.26	13.32
P_5 C-Sb-Sb-W(Cl)	1.33	2.78	2.51	2.33	....	8.96	2.03	1.56	1.12	4.71	13.67
P_6 C-Sb-Sb-W-Cl	1.83	2.56	2.22	2.18	....	8.79	.93	1.68	1.65	4.26	13.05
P_7 W-Sb	....	4.02	2.48	....	....	6.50	.54	1.44	.72	2.70	9.20
P_8 C-C-Sb-W-Cl	3.45	1.17	1.92	2.18	....	8.72	.24	.67	.76	1.67	10.39
P_9 C-C-Sb-W(Cl)	3.56	1.99	2.31	2.33	....	10.19	1.62	1.45	.88	3.95	14.14
P_{10} C-Sb-Rt-Rt-Rt	2.47	1.61	....	....	2.16	6.24	1.20	.54	.56	2.30	8.54
P_{11} C-Rt-Rt-Rt-Sb	1.50	1.29	....	....	2.16	4.95	.96	.20	.30	1.46	6.41
P_{12} Sb-W-Rt-Rt-Rt-Rt	....	.88	1.14	....	2.40	4.42	.73	.19	.10	1.02	5.44
P_{13} C-C-C	7.93	....	....	....	....	7.93	3.43	2.82	2.18	8.43	16.36
P_{14} Sb-Sb-Sb	....	7.72	....	....	....	7.72	....	....	.48	.48	8.20

Appendix Tables 2 and 3. Prices for fertilizer materials were estimated on the basis of an informal survey of fertilizer dealers in 1958.[1]

Variable costs for each crop, other than for fertilizer, are indicated in Appendix Table 4. The variable cost figures used in the model reflect an adjustment for differences between yields reported in the survey and in the Farm Bureau Farm Management Service records (Appendix Table 1) which were used as the basic source for the estimates. The 1954 and 1958 totals of variable costs per rotation acre, including fertilizer, are shown for the east-central area in Table 7, and for the southern area in Table 8.

In developing the model it was important to so adjust yields in each area that they are comparable between 1954 and 1958 in terms of all growing conditions other than for differences in fertilizer programs found in the survey. Yields reported in 1958 by the University of Illinois on experimental plots, with the same soil types as found on the survey farms, were arranged in sequences and plotted according to fertilizer nutrients and application levels. Application rates in the experimental data were then matched with those of the survey data, and the response from fertilizer inputs in 1958 for each crop was calculated over the application ranges, comparable with those found in the survey data.[2]

Production for 1954 was adjusted on the basis of 1958 production conditions other than for fertilizer. Estimates were made through adjustment of 1958 yields by the net response to the change in fertilizer inputs. Thus:

$$\hat{Y} = a - (b_{ij}\Delta N)$$

Where $\hat{Y}$ = output adjusted for 1954.

a = 1958 yield in survey data.

b_{ij} = the marginal product from 1 lb. of nutrient (obtained from 1958 experiments in which only that nutrient was varied, and at rates comparable with survey rates); i refers to the nutrient combination on which observations were taken and j refers to the nutrient varied.

ΔN = difference in level of nutrient applied in 1958 and 1954 in survey data.

[1] Five randomly selected fertilizer dealers and distributors from manufacturers' lists in each soil area were interviewed to determine volumes and retail prices of each type of plant food sold. No important difference in nutrient prices was found between the soil areas. Nutrient costs per pound were as follows: nitrogen 12 cents, phosphoric acid 8 cents, and potash 4 cents.

[2] Experimental yield data for these procedures were furnished from unpublished materials of the Agronomy Department, University of Illinois, by L. B. Miller.

These yields are actually rotation-acre[1] yields (total output from X_j) in which the output of each crop has been adjusted. Changes in nonfertilizer factors other than weather were assumed to have consequences that were (a) the same between crop sequences within each area, and (b) the same between areas with respect to all crop sequences.

By varying crop prices the model was used to solve for a series of optimum cropping systems for farms in each soil area under fertilizer programs typical of 1954 and 1958 conditions. In Tables 9 and 10, these systems are described for the Flanagan and Cisne areas, respectively. By referring to these tables, results of the price variation are displayed in Figs. 4 through 6 for the Flanagan area and in Figs. 7 and 8 for the Cisne area.

Table 9. — Land Use and Composition of Optimum Farming Systems on Flanagan Soils

System	Land use				System composition
	Corn	Soybeans	Wheat	Oats	
	(acres)				
A........	50.67	50.67	50.67	.....	152a C-Sb-W
B........	67.62	50.67	33.71	.....	101a C-Sb-W; 51a C-C-Sb
C........	78.35	50.64	.....	22.93	83.21a C-C-Sb; 68.79a C-Sb-O
D........	50.67	50.67	.....	50.67	152a C-Sb-O
E........	66.09	42.95	42.95	.....	128.86a C-Sb-W; 23.14a C-C-C
F........	80.35	40.17	.....	31.48	125.93a C-Sb-C-O; 26.07a C-C-Sb
G........	81.23	35.39	.....	35.38	141.54a C-Sb-C-O; 10.46a C-C-C

INTERPRETATION OF RESULTS

"Price maps," which are shown in Figs. 4 through 8, are used to interpret the data. On the axes are shown prices for crops, wheat and corn, for example, in Fig. 4. In this figure, the various lines delineate combinations of the two crop prices as "boundaries" for systems described in Table 9. Thus we find that system A (all tilled land in a corn-soybeans-wheat rotation) was optimal under the 1954 fertilizer program for corn priced in a range from \$.60 to \$1.35 per bushel, and wheat from \$1.30 to \$2.50 per bushel. To achieve these results, soybean

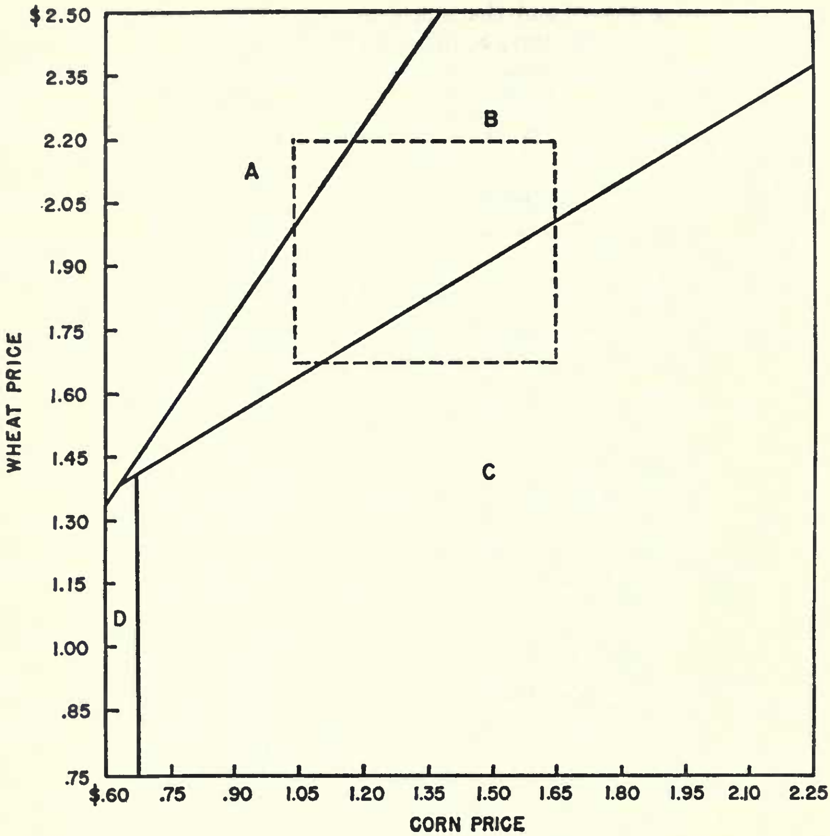
[1] Rotation acre is defined as one acre of land growing the same percentage of each crop as appears in the rotation.

Table 10. — Land Use and Composition of Optimum Farming Systems on Cisne Soils

System	Land use: Corn	Soybeans	Wheat	Redtop	Clover	System composition
	(acres)					
A........	.....	123.0	.....	.....	.....	123a Sb-Sb-Sb
B........	66.36	56.64	.....	.....	.....	56.64a Sb-Sb-Sb; 66.36a C-C-C
C........	73.95	12.26	.....	36.79	.....	61.31a C-Sb-Rt-Rt-Rt; 61.69a C-C-C
D........	.....	20.50	20.50	82.00	.....	123a Sb-W-Rt-Rt-Rt-Rt
E........	69.27	8.96	8.96	35.81	.....	53.73a Sb-W-Rt-Rt-Rt-Rt; 69.27a C-C-C
F........	41.00	41.00	41.00	.....	.....	123a C-Sb-W
G........	53.26	34.87	34.87	.....	.....	104.60a C-Sb-W; 18.40a C-C-C
H........	39.11	83.89	.....	.....	.....	78.22a C-Sb; 44.77a Sb-Sb-Sb
I........	61.50	61.50	.....	.....	.....	123a C-Sb
J........	37.97	66.05	18.98	.....	.....	75.93a C-C-Sb-W(Cl); 47.07a Sb-Sb-Sb
K........	55.87	39.19	27.94	.....	.....	111.74a C-C-Sb-W(Cl); 11.26a Sb-Sb-Sb
L........	61.50	48.55	12.95	.....	.....	51.79a C-C-Sb-W(Cl); 71.21a C-Sb
M........	66.37	56.63	.....	.....	.....	9.73a C-C-C; 113.27a C-Sb
N........	57.35	28.67	28.67	.....	8.31	81.46a C-C-Sb-W(Cl); 41.54a C-C-Sb-W-Cl
P........	56.34	29.89	29.89	6.88	.....	112.68a C-C-Sb-W(Cl); 10.32a Sb-W-Rt-Rt-Rt-Rt
Q........	63.09	19.97	19.97	.....	19.97	23.12a C-C-C; 99.87a C-C-Sb-W-Cl
R........	24.60	24.60	.....	73.80	.....	123a C-Sb-Rt-Rt-Rt

prices were held constant at \$2.35 per bushel. Zones encompassing price combinations that may be considered relevant (e.g., approximate range of elevator prices 1953-57) are outlined on the price map by a dotted line. Specification of these price zones narrows the area of investigation and simplifies the derivation of the price maps shown in Figs. 5 through 8. In these figures, a more comprehensive picture is given because the prices are varied for three crops.

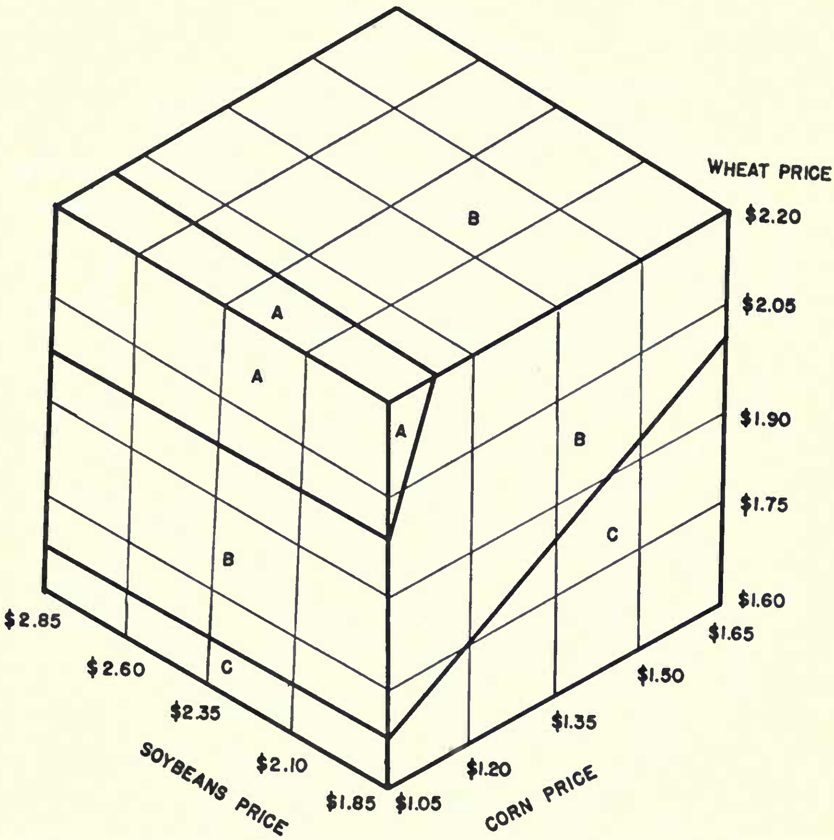
Fig. 5 shows the optimum crop relationships in the Flanagan soil area for 1954. A vertical slice through this figure perpendicular to the soybean price axis is the counterpart of a corn-wheat price map. (The area within the relevant price zone on Fig. 4 corresponds to a vertical slice through Fig. 5 at the \$2.35 soybean price.) Comparisons between two price maps (e.g., Figs. 5 and 6) give an indication of the shifts in rotation composition that accompany the changes in fertilizer programs.



Corn-wheat price map for the modal farm on Flanagan soils. Approximate range of elevator prices are outlined by dotted lines. (Fig. 4)

Table 11. — Percentages of Available Acreages Allocated to Specific Crops in Optimal and Actual Programs by Time Period

	Corn		Soybeans		Small grain	
	Optimal	Actual	Optimal	Actual	Optimal	Actual
Flanagan soil area						
1958	51.6	44.80	33.3	37.29	15.1	17.19
1954	44.5	45.45	33.3	32.30	22.2	22.25
Differences	7.1	−.65	0	4.99	−7.1	−5.06
Cisne soil area						
1958	54.0	23.37	46.0	55.74	0	20.89
1954	45.4	34.08	31.9	41.63	22.7	24.29
Differences	8.6	−10.71	14.1	14.11	−22.7	−3.40



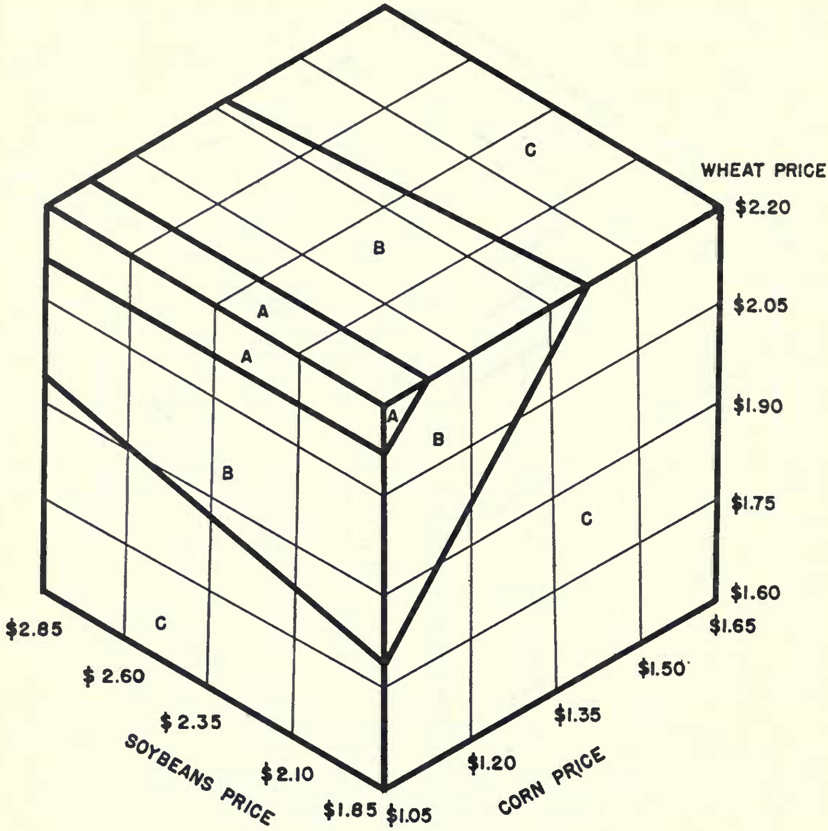
The effect of price on optimum cropping systems on Flanagan soils, 1954. (See Table 9 for definition of systems.) (Fig. 5)

Shifts in the Flanagan soil area

On Flanagan soils the basic cropping shift was from system B in 1954 to system C in 1958. (Table 9 indicates the composition of each of the optima found for the Flanagan soil area model.) As newer fertilizer programs were adopted, corn was substituted for small grain[1] in the Flanagan soil area. Of the 152 acres available for competitive cropping on the Flanagan soil modal farm, the optimum farming system shifted from use of approximately two-thirds of the land in the corn-soybeans-wheat sequence (P_7),[2] and one-third in corn-corn-

[1] Small grain units are in wheat-equivalent units (one bushel of oats = 0.3177 bushel of wheat-equivalent).

[2] Designations in parentheses refer to activity numbers indicated in Appendix Tables 5 and 6.



The effect of price on optimum cropping systems on Flanagan soils, 1958. (See Table 9 for definition of systems.) (Fig. 6)

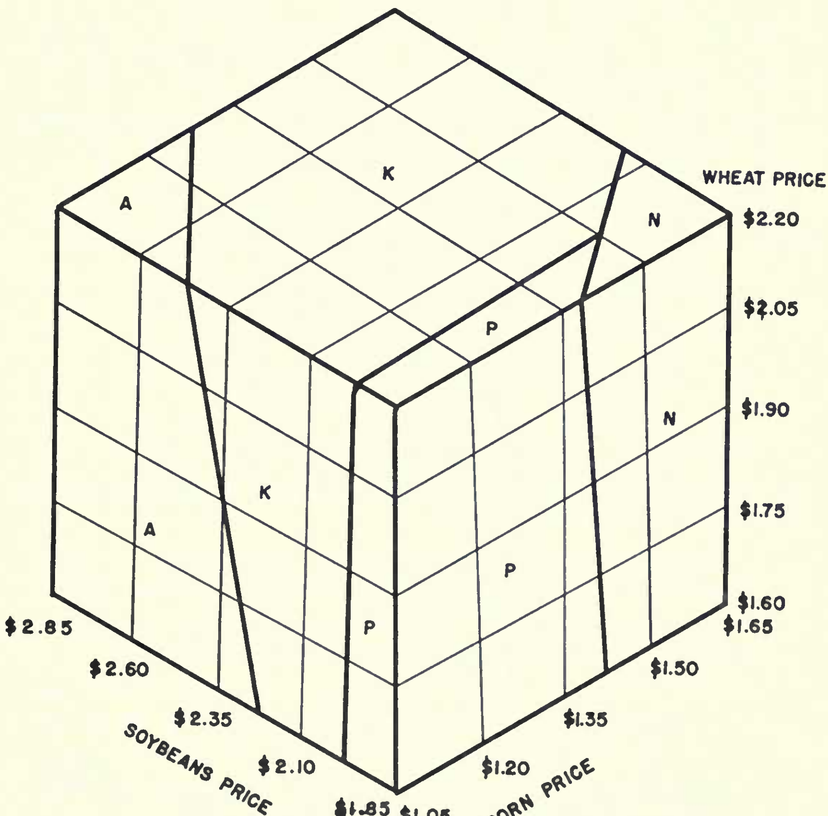
soybeans (P_{12}), to one with 55 percent of the land in corn-corn-soybeans (P_{12}), and 45 percent in a corn-soybeans-oats sequence (P_6). Thus corn acreage increased by about 11 acres or 7.1 percent. A corresponding reduction of 11 acres (7.1 percent) occurred in small grain. Soybean acreage remained virtually unchanged. (See Table 11.)

Analysis of the farm survey data for all sample farms in the soil area disclosed the acreage allocation designated in Table 11 as "actual" for each time period.

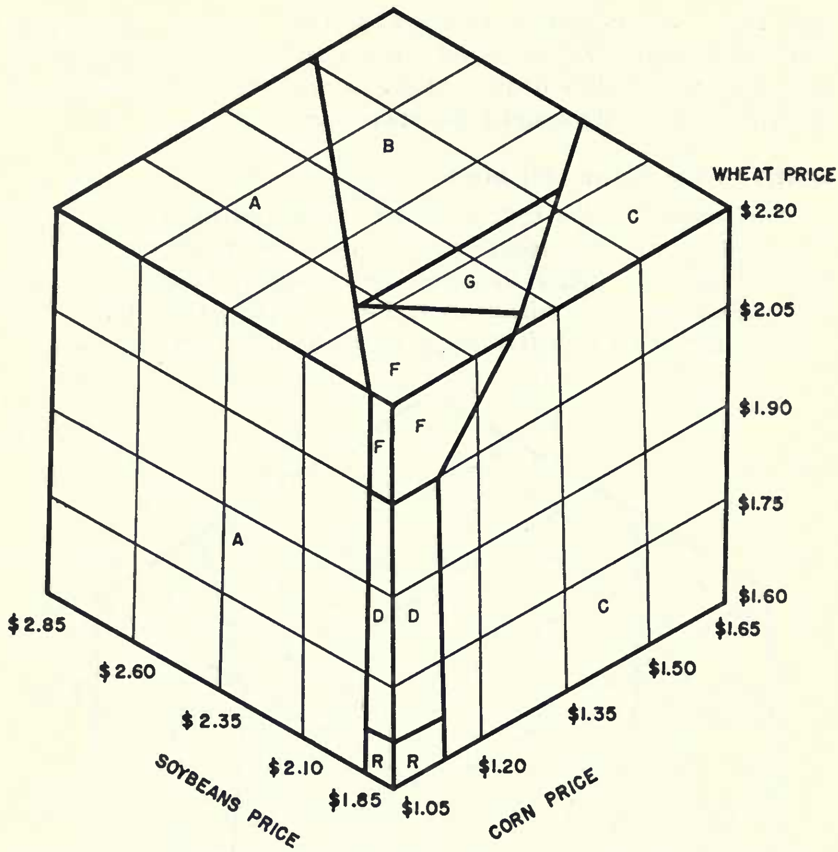
The "optimal" and "actual" figures are not directly comparable, since the optimal programs were optimum at crop-price ratios representing the five-year average of price in 1954-1958. The observed allocations (actual) were associated with existing price ratios in 1954 and 1958. The optimal programs also reflect production conditions adjusted for 1954 rather than existent conditions. These facts may account for the failure of the actual allocation percentages to coincide more closely with the optimal allocation figures.

Shifts in the Cisne soil area

An analysis of the optima similar to that on the Flanagan soils was made of the crop relationships in the Cisne soils area (compare Figs. 7 and 8). Fig. 7 indicates the optima at various crop-price relationships on Cisne soils in 1954. Fig. 8 indicates the corresponding optima in 1958, a newer fertilizer program having been adopted.



The effect of price on optimum cropping systems on Cisne soils, 1954. (See Table 10 for definition of systems.) (Fig. 7)



The effect of price on optimum cropping systems on Cisne soils, 1958. (See Table 10 for definition of systems.) (Fig. 8)

On the Cisne soils the basic shifts in optima were from system K in 1954 to system B in 1958. (Table 10 indicates the composition of each of the optima found for the Cisne soils area model.)

With the change in fertilizer program, small grain was replaced largely by soybeans and to a lesser extent by corn. The clover catch-crop was eliminated in the optimal plan. Of the 123 acres available for competitive cropping on the farm modal for Cisne soils, the optimum farming system within the price zones designated shifted from use of slightly over 90 percent of the land in corn-corn-soybeans-wheat (clover catch), (P_9),[1] and about 10 percent in a continuous soybeans

[1] Designations in parentheses refer to activity numbers indicated in Appendix Tables 7 and 8.

sequence (P_{14}), to one of about 45 percent in continuous soybeans (P_{14}), and 55 percent in continuous corn (P_{13}). This shift in the optimal plan represents an over-all increase in soybean acreage of about 16 acres (14.1 percent), and an increase of about 10 acres (8.6 percent) in corn. Small grains were reduced by 27 acres (22.7 percent). (See Table 11.)

The "actual" figures in Table 11 come from an analysis of the survey data in the soil area. The characteristics of these figures, which do not permit direct comparison, were noted previously. Wheat was favored more in the 1958 price ratios than in the ratios from the 1954-1958 average. This fact may account for the failure of the actual small grains percentage to coincide more closely with the optimal figure.

Shifts between soil areas

The relative shifts in crop acreages between soil areas are illustrated in Table 12. The shares of the allocable acreages devoted to each of the important crops in the two soil areas in the 1954 optima were quite similar. There was less than 1.5 percent difference between the areas in the percentages of available acreages devoted to specific crops in the optimum programs. In the 1958 optima, with the change in fertilizer programs, this situation had changed considerably.

The proportion of the allocable acreage devoted to corn in the 1958 optima was 2.4 percent more in the Cisne area than in the Flanagan soil area. The really significant shifts took place in soybeans and small grains. The proportion of the acreage allocated to soybeans in the 1958 optima was 12.7 percent more in the Cisne than in the Flanagan area. Conversely, the proportion of the acreage devoted to small grains was

Table 12. — Shifts Between Soil Areas in the Relative Importance of Specified Crops in the Optimal Program: by Percentage of Allocable Acreage, by Time Period

	Corn	Soybeans	Small grain	Total
	1954			
Cisne	45.4	31.9	22.7	100.0
Flanagan	44.5	33.3	22.2	100.0
Differences	.9	−1.4	.5	0
	1958			
Cisne	54.0	46.0	0	100.0
Flanagan	51.6	33.3	15.1	100.0
Differences	2.4	12.7	−15.1	0

Table 13. — Changes in Fertilizer Applications Accompanying Shifts in Optimum Cropping Programs

Program	N	P_2O_5	K_2O
		(pounds)	
Flanagan soils			
1958	5,427	3,241	2,465
1954	3,612	2,483	2,483
Difference	1,815	758	−18
Difference per acre	11.94	4.99	−.118
Cisne soils			
1958	1,986	2,225	3,889
1954	375	2,041	1,493
Difference	1,611	184	2,396
Difference per acre	13.10	1.50	19.48

15.1 percent less in the Cisne than in the Flanagan soil area. Thus the change in fertilizer program reduced the relative advantage of Flanagan soils in terms of both corn and soybeans, but especially soybeans.

Changes in fertilizer applications

Basic changes in fertilizer nutrient applications accompanying shifts in cropping programs are indicated in Table 13, for Flanagan and Cisne soil areas respectively. Nutrients appear to be applied on Flanagan soils at a fairly high point on the response curve (at a relatively higher or flatter point) when compared with the Cisne soil curve. This may help explain why marginal returns from fertilizer on Flanagan soils are commonly thought to be lower than on Cisne soils. Also, it is consistent with the observation that there was less change on Flanagan soils than on Cisne soils in crop production accompanying the change in fertilizer application. (Compare the data in Table 11.)

Fertilizer changes per acre over the five-year period were quite similar in both areas for nitrogen application. Moderate differences occurred in application of P_2O_5 (4 pounds), and considerable difference was noted in application of K_2O (19 pounds). Cisne soils are notably low in K_2O. Crops on these soils are considered very responsive to potassium. The increase in per-acre applications of K_2O is evidence of recognition and reduction of this nutrient deficiency.

INTERPRETATION OF SUBSTITUTION RELATIONSHIPS

The following general procedure was carried out to determine the marginal rates of substitution between crops in both soil areas for each time period:

The production of every crop was ascertained in each of the optimum cropping systems by using the solutions described above.[1] These production data were then related by fitting equations of the form:

$$Ax + By + Cz + Dv + E = 0$$

Where x = production of soybeans in bushels.

y = production of small grains in bushels.

z = production of corn in bushels.

v = production of redtop seed on Cisne soils in pounds.

The partial derivatives of the resulting functions were then taken with respect to each of the crops and substitution relationships were derived.

Since there was no basis for deciding which of the variables should be considered dependent in the functions (i.e., whether to minimize the vertical or the horizontal deviation from the fitted function), an averaging process was used to derive a single value for the relationships. This process involved the following types of manipulations:

$$\frac{\dfrac{\delta z}{\delta x} + \dfrac{1}{\dfrac{\delta x}{\delta z}}}{2} = \frac{\delta z^*}{\delta x} \quad \text{(Substitution of soybeans for corn)}$$

$$\frac{\dfrac{\delta z}{\delta y} + \dfrac{1}{\dfrac{\delta y}{\delta z}}}{2} = \frac{\delta z^*}{\delta y} \quad \text{(Substitution of small grains for corn)}$$

$$\frac{\dfrac{\delta x}{\delta y} + \dfrac{1}{\dfrac{\delta y}{\delta x}}}{2} = \frac{\delta x^*}{\delta y} \quad \text{(Substitution of small grains for soybeans)}$$

Substitutions on Flanagan soils

The production data for the Flanagan soil area were related by fitting the equation of a plane for each time period. The equations of the plane determined by three points were found by substituting in turn the coordinates of the points in the equation

$$Ax + By + Cz + D = 0,$$

---

* Designates computed marginal rate of substitution. The use of "2" in the denominators presumes a constant marginal rate of substitution over the narrow range here considered.

[1] The compositions of the systems indicated in Figures 5 through 8 are designated in Tables 9 and 10 for the Flanagan and Cisne soils areas respectively. Crop yields per rotation-acre are indicated in Appendix Tables 5 through 8.

thus obtaining three equations to solve for three of the constants in terms of the fourth.[1]

The equations fitted for the 1954 relationships on Flanagan soils were:

$$- 13.762x + 2.042y + z + 12644.91 = 0$$
or
$$z = -12,644.91 + 13.762x - 2.042y;$$
$$- 6.741x + y + 0.490z + 6,193.607 = 0 \qquad N = 3$$
or
$$y = -6,193.607 + 6.741x - 0.490z;$$
$$x - 0.148y - 0.073z + 2315.030 = 0$$
or
$$x = -2315.030 + 0.148y + 0.073z.$$

The averaging process discussed above produced the following crop relationships for Flanagan soils in 1954:[2]

$$\frac{\delta \overset{*}{z}}{\delta x} = +13.730$$

$$\frac{\delta \overset{*}{z}}{\delta y} = -2.041$$

$$\frac{\delta \overset{*}{x}}{\delta y} = +0.148$$

The equations fitted for the 1958 relationships on Flanagan soils were:

$$+ 8.276x + 2.451y + z - 24,323.91 = 0$$
or
$$z = 24,323.91 - 8.276x - 2.451y;$$
$$+ 3.377x + y + 0.408z - 9926.33 = 0 \qquad N = 3$$
or
$$y = 9926.33 - 3.377x - 0.408z;$$
$$x + 0.296y + 0.121z - 2939.07 = 0$$
or
$$x = 2939.07 - 0.296y - 0.121z.$$

The averaging process produced the following relationships for Flanagan soils in 1958:

[1] For detail on calculation of gradients for a plane see, Allen, R. G. D., *Mathematical Analysis for Economists*, London, 1956, p. 305 ff.

[2] The slight differences in the Flanagan soil area between the computed MRS's and the single partial derivatives from the fitted equations are due to rounding.

$$\frac{\delta \overset{*}{z}}{\delta x} = -8.270$$

$$\frac{\delta \overset{*}{z}}{\delta y} = -2.451$$

$$\frac{\delta \overset{*}{x}}{\delta y} = -0.296$$

Under 1954 fertilizer conditions, at the optimum, using prices in the indicated zone, the computed marginal rate of substitution of soybeans for corn was + 13.73. With the adoption of the newer fertilizer program the MRS of soybeans for corn was changed to − 8.27. (See Table 14 for a summary of marginal rates of substitutions stated as reciprocals.)

The improved fertilizer program seemingly caused a movement of the equilibrium into the competitive range. The direction of this shift might have been anticipated. Although soybeans are classified as a legume, their soil-building qualities are debatable. Their action in these relationships, however, is in the nature of soil-building legumes. The soil-building qualities are replaced by commercial nutrients in the shift.

The computed MRS of small grain for corn in the earlier period (1954) was − 2.041. Thus about 2 bushels of small grain were substituted at an optimum for 1 bushel of corn in the use of resources common to both crops, including fertilizer. With the adoption of the newer fertilizer program, this crop relationship for 1958 changed to − 2.45. The change in the fertilizer program would be interpreted, in this case, as having caused a decrease in the rate at which small grains

Table 14. — Computed Marginal Rates of Substitution Between Crops, by Soil Area and Time Period, and Changes Over Time

	1954	1958	Absolute change
Flanagan soil area			
MRS corn for soybeans	+.073	−.121	.194
MRS corn for small grains	−.490	−.408	.082
MRS small grains for soybeans	+.148	−.296	.444
Cisne soil area			
MRS corn for soybeans	−.635	+.252	.887
MRS corn for small grains	−.785	−.155	.630
MRS small grains for soybeans	−.796	+1.833	2.629

substitute for corn. With this decrease, corn should tend to replace small grain at the same price ratios.

The directional change of the MRS of small grain for soybeans paralleled the corn-soybeans relationship on these soils. This computed coefficient shifted from + 0.148 in 1954 to − 0.296 in 1958. The change in the fertilizer program seemingly caused a shift of the equilibrium into the competitive range.

The effects of the change in the fertilizer program appear to be more marked on the corn-soybeans relationship than on the corn-small grain relationship. The competitive positions of corn and soybeans seem to be favored relative to that of small grains. The results of this analysis offer a plausible explanation for the shift from small grain to corn. This was noted above in the comparison of optimum cropping programs.

Substitutions on Cisne soils

The production data for the Cisne soil area were related by fitting equations similar to those for the Flanagan soil area. The procedure differed somewhat since more optima were generated in the solutions (the optima were less stable in response to crop price change than those of the Flanagan soil area). In the Cisne soil data, six optima were generated in each time period on four variables. Equations were then fitted by the method of least squares.

The equations fitted for the 1954 relationships on Cisne soils were:

$$1.498x + 0.753y + z + 0.903v - 4{,}337.15 = 0 \quad \text{or}$$
$$z = 4{,}337.15 - 1.498x - 0.753y - 0.903v;$$

$$x + 0.500y + 0.605z + 0.650v - 2{,}798.31 = 0 \quad \text{or}$$
$$x = 2{,}798.31 - 0.500y - 0.605z - 0.650v;$$

$$0.916x + y + 0.557z + 0.266v - 2{,}785.4 = 0 \quad \text{or}$$
$$y = 2{,}785.40 - 0.916x - 0.557z - 0.266v.$$

The marginal rates of substitution derived from the averaging process on Cisne soil data in 1954 were as follows:

$$\frac{\delta \overset{*}{z}}{\delta x} = -1.576$$

$$\frac{\delta \overset{*}{z}}{\delta y} = -1.274$$

$$\frac{\delta \overset{*}{x}}{\delta y} = -0.796$$

The equations fitted for the 1958 relationships on Cisne soils were:

$$-1.759x + 0.701y + z + 0.091v - 1{,}307.18 = 0 \quad \text{or}$$
$$z = 1{,}307.18 + 1.759x - 0.701y - 0.091v;$$

$$x - 0.230y - 0.162z + 0.011v - 246.79 = 0 \quad \text{or}$$
$$x = 246.79 + 0.230y + 0.162z - 0.011v;$$

$$-0.291x + y + 0.082z + 0.007v - 265.25 = 0 \quad \text{or}$$
$$y = 265.25 + 0.291x - 0.082z - 0.007v.$$

The marginal rates of substitution derived from the averaging process with the 1958 Cisne soil relationships were as follows:

$$\frac{\delta \overset{*}{z}}{\delta x} = +3.966$$

$$\frac{\delta \overset{*}{z}}{\delta y} = -6.448$$

$$\frac{\delta \overset{*}{x}}{\delta y} = +1.833$$

With the 1954 fertilizer program, the computed marginal rate of substitution of soybeans for corn was − 1.576 within the indicated price zone. (See Table 14 for a summary of marginal rates of substitution.) With the adoption of the newer fertilizer programs, the MRS of soybeans for corn was changed to + 3.966. The change in the fertilizer program apparently caused the equilibrium to shift into the complementary range.

The computed MRS of small grain for corn in 1954 was − 1.274. With the adoption of improved fertilizer programs, this crop relationship changed to − 6.448 in 1958. As on Flanagan soils the effect of changes in the fertilizer program was to decrease the rate at which small grains substitute for corn. Thus corn should tend to replace small grain at the same price ratio. As was also noted on Flanagan soils, the greater relative change resulting from the adoption of improved fertilizer programs occurred in the corn-soybeans relationship.

Unlike the MRS of small grain for soybeans on Flanagan soils, this relationship on Cisne soils paralleled the shift in MRS of soybeans for corn on Cisne soils. The computed MRS of small grain for soybeans shifted from − 0.796 on Cisne soils in 1954 to + 1.833 in 1958.

Comparisons between soil areas

Comparisons between soil areas of differences over time between marginal rates of crop substitution can be observed in Table 14. These

Table 15. — Comparisons Between Soil Areas of Computed Marginal Substitution Rates, Between Crops, by Time Period

	1954	1958
MRS corn for soybeans		
Flanagan	+.073	−.121
Cisne	−.635	+.252
Difference	.708	.373
MRS corn for small grain		
Flanagan	−.490	−.408
Cisne	−.785	−.155
Difference	.295	.253
MRS small grains for soybeans		
Flanagan	+.148	−.296
Cisne	−.796	+1.833
Difference	.944	2.129

comparisons support one of the early hypotheses in this study, namely, that there has been a change over time in the marginal rates of substitution between crops in both east-central and south-central Illinois. The effects of the changes in the fertilizer program have been considerably greater in the Cisne soil area than on Flanagan soils in all the substitution relationships examined.

Comparison of differences between soil areas in the marginal rates of substitution between crops (Table 15), supports another of the hypotheses in this study, namely, that there was a difference between east-central and south-central Illinois in the marginal rates of substitution between crops in 1954 and 1958.

Apparently the adoption of improved fertilizer programs has reduced the differences between the Flanagan soil area and the Cisne soil area in the crop substitution relationships involving corn.

SUMMARY AND CONCLUSIONS

The primary objectives of this study were (1) using two Illinois soils areas, to isolate the effects of commercial fertilizer on crop selection, and (2) to develop an analytic model that may be used in investigating the effects of technological developments.

Data taken from a sample of contrasting soil types on 182 farms (96 in east-central Illinois and 86 in south-central Illinois) were used to establish resource restrictions and to define production opportunities in models for the two contrasting soil areas at two points in time. Profit-maximizing optima were generated with these models. From optimal responses to price variation, marginal rates of substitution between crops were calculated in each area at each time period and comparisons were made within areas over time. In all cases every effort was made to eliminate causes of variation other than those due to fertilizer use.

The results of this study support the hypothesis that there has been a change over a period of time in the marginal rates of substitution between crops in east-central and south-central Illinois. The study revealed the following about the changes in the use of commercial fertilizer:

1. The effects of the changes in fertilizer programs on crop relationships have been considerably greater in the Cisne soil area than on Flanagan soils.

2. The adoption of improved fertilizer programs has reduced the differences between the Flanagan soil area and the Cisne soil area in the crop substitution relationships involving corn.

3. In both the Flanagan and Cisne soil areas the effects of the changes in fertilizer programs appeared to be more marked on the corn-soybeans relationships than on the corn-small grain relationships.

4. In both the Flanagan and Cisne soil areas the competitive positions of corn and soybeans seemed to be favored relative to that of small grains.

5. The procedures employed in the study were successful in isolating the effects of commercial fertilizer use on the crop substitution relationships. There were no results derived which would contradict this conclusion.

6. The analytic model developed here in analyzing the effects of commercial fertilizer appears adaptable to the investigation of the effects of similar technological developments. The effects of irrigation, herbicides, and insecticides could certainly be analyzed in a similar analytic framework.

Differences between areas and shifts in the production possibilities curves may have significant implications. With the change in fertilizer programs discussed in this study, the advantage enjoyed by farmers on good soils decreased relative to that of farmers on poorer soils. The consequences of these changes in the competitive positions of farmers in relatively large geographic areas may have an important influence on regional specialization. Between the areas investigated in this study the geographic boundaries may become less distinct, and the differences in resource values less pronounced.

APPENDIX TABLES

Appendix Table 1. — Feed Requirements for Classes of Livestock Found on Modal Farms and Total Requirements by Areas[a]

Kind	Production	Corn equivalent	Hay equivalent	Protein
		(bushels)	(tons)	(pounds)
Dairy cow........	(8,500 pounds milk, 400 pounds meat)	50	4.5	600
Beef cow.........	(480 pounds meat)	10	2.0	....
Ewe and lamb....	(9 pounds wool, 93 pounds meat)	2.5	.2	10
Sow—2 litters....	(3,400 pounds pork[b])	229	....	1,635
Sow—1 litter.....	(1,600 pounds pork)	110	....	740
Total feed requirement for livestock complement, east-central area..........		607	12.03	4,297
Total feed requirement for livestock complement, south-central area.........		568	21.55	3,840

[a] Hinton, R. A., "Farm Management Manual," Dept. Agr. Econ., Univ. Ill., Mimeograph AE-3349, Jan. 1959, p. 10.
[b] Adjustments made from table work sheets for larger average litter size.

Appendix Table 2. — Nutrient Application Rates Indicated in Survey by Crops, in Different Rotations, East-Central Illinois, 1954 and 1958

Crop rotation		1958 Fertilizer applied per acre			Actual yield	1954 Fertilizer applied per acre			Adjusted yield
		N	P_2O_5	K_2O		N	P_2O_5	K_2O	
		(pounds)			(bushels)	(pounds)			(bushels)
(1) C-Sb-W-Cl									
	C	0	0	0	76.76	0	0	0	73.13
	Sb	0	0	0	32.95	0	0	0	31.62
	W	34.30	40.54	41.35	49.08	27.37	27.37	27.37	44.11
	Cl	0	0	0	0	0	0	0	0
(2) C-Sb-O-Cl									
	C	62.71	32.68	43.33	91.07	0	0	0	61.68
	Sb	0	0	0	34.00	0	0	0	32.63
	O	0	0	0	77.11	0	0	0	74.90
	Cl	0	0	0	0	0	0	0	0
(3) C-Sb-W(Cl)									
	C	43.85	30.48	28.05	91.00	54.20	25.26	25.26	90.83
	Sb	0	0	0	31.55	0	0	0	30.28
	W	20.20	35.31	35.31	45.26	0	0	0	37.14
(4) C-Sb-O(Cl)									
	C	0	0	0	75.43	0	0	0	71.86
	Sb	0	0	0	29.70	0	0	0	28.50
	O	0	0	0	53.94	0	0	0	52.39
(5) C-Sb									
	C	54.49	43.92	23.57	85.32	0	0	0	59.49
	Sb	0	0	0	31.34	0	0	0	30.08
(6) C-Sb-O									
	C	52.82	27.65	27.65	120.00	0	0	0	93.19
	Sb	0	0	0	36.47	0	0	0	35.00
	O	0	0	0	75.21	0	0	0	73.05
(7) C-Sb-W									
	C	56.71	39.52	31.05	103.70	35.06	34.58	34.58	90.13
	Sb	0	0	0	31.63	0	0	0	30.35
	W	34.34	40.21	46.88	46.77	57.00	24.00	24.00	52.88

(Table is concluded on next page)

Appendix Table 2. — Concluded

Crop rotation		1958 Fertilizer applied per acre N	P₂O₅	K₂O	Actual yield	1954 Fertilizer applied per acre N	P₂O₅	K₂O	Adjusted yield
		(pounds)			(bushels)	(pounds)			(bushels)
(8) C-O-Cl									
	C	32.00	46.35	46.35	87.03	70.00	0	0	98.11
	O	0	0	0	60.00	0	0	0	58.28
	Cl	0	0	0	0	0	0	0	0
(9) C-C-Sb-O-Cl									
	C	53.00	68.00	120.00	100.00	0	0	0	74.07
	C	60.91	20.00	60.00	101.52	0	0	0	72.36
	Sb	0	0	0	30.36	0	0	0	29.14
	O	0	0	0	74.55	0	0	0	72.41
	Cl	0	0	0	0	0	0	0	0
(10) C-C-Sb-W(Cl)									
	C	60.00	70.00	70.00	98.33	0	0	0	69.68
	C	23.70	30.13	42.14	83.61	31.00	23.33	36.67	82.57
	Sb	0	0	0	34.70	0	0	0	33.30
	W	46.43	35.67	35.67	38.37	23.41	37.93	37.93	29.52
(11) C-C-Sb-O(Cl)									
	C	52.96	30.74	30.74	93.06	0	0	0	67.47
	C	66.00	0	0	90.00	0	0	0	59.34
	Sb	0	0	0	32.81	0	0	0	31.49
	O	0	0	0	64.29	0	0	0	62.44
(12) C-C-Sb									
	C	54.00	49.00	36.00	98.50	0	0	0	72.24
	C	98.00	45.00	30.00	105.00	30.00	30.00	30.00	72.83
	Sb	0	0	0	33.42	0	0	0	32.07
(13) C-C-O-Cl									
	C	0	0	0	94.47	0	0	0	90.01
	C	73.03	0	60.00	82.08	56.67	40.00	40.00	71.65
	O	0	0	0	55.60	0	0	0	54.00
	Cl	0	0	0	0	0	0	0	0
(14) C-C-C									
	C	19.20	15.20	19.20	69.01	0	0	0	61.36
(15) C-Sb-C-O									
	C	44.23	25.75	12.88	90.64	42.69	26.51	26.51	83.74
	Sb	0	0	0	26.73	0	0	0	25.65
	C	39.00	24.00	12.00	100.00	36.00	12.00	12.00	93.47
	O	0	0	0	33.33	3.00	12.00	12.00	33.87

Appendix Table 3. — Nutrient Application Rates Indicated in Survey by Crops, in Different Rotations, South-Central Illinois, 1954 and 1958

Crop rotation		1958 Fertilizer applied per acre N	P₂O₅	K₂O	Actual yield	1954 Fertilizer applied per acre N	P₂O₅	K₂O	Adjusted yield
		(pounds)			(bushels)	(pounds)			(bushels)
(1) C-Sb-W-Cl									
	C	22.13	35.17	40.51	44.13	13.04	23.14	37.54	36.96
	Sb	0	0	0	26.37	0	0	0	24.96
	W	18.86	52.64	24.54	27.68	17.57	26.08	26.74	25.89
	Cl	0	0	0	0	0	0	0	0
(2) C-Sb-W(Cl)									
	C	23.16	38.93	67.34	42.09	26.00	72.00	120.00	39.85
	Sb	0	0	0	23.39	0	0	141.82	22.17
	W	16.70	41.34	80.32	25.62	33.00	45.00	0	29.79
(3) C-Sb									
	C	9.43	24.22	39.42	42.90	6.80	23.38	30.09	38.41
	Sb	0	0	0	23.55	0	0	0	23.22
(4) C-Sb-W									
	C	34.90	40.02	41.76	42.31	24.14	27.14	27.14	34.62
	Sb	0	0	0	24.06	0	0	0	22.81
	W	25.04	24.31	44.74	30.96	34.89	30.89	30.89	32.71
(5) C-Sb-Sb-W(Cl)									
	C	33.07	17.03	48.66	31.82	36.52	24.00	24.00	30.65
	Sb	0	0	0	17.00	0	0	0	16.11
	Sb	0	0	0	17.27	0	0	0	16.37
	W	34.54	60.07	63.18	33.48	0	0	0	20.32

(Table is concluded on next page)

Appendix Table 3. — Concluded

Crop rotation		1958 Fertilizer applied per acre N	P₂O₅	K₂O	Actual yield	1954 Fertilizer applied per acre N	P₂O₅	K₂O	Adjusted yield
		(pounds)			(bushels)	(pounds)			(bushels)
(6) C-Sb-Sb-W-Cl									
	C	6.00	24.00	86.07	54.72	8.00	32.00	32.00	51.13
	Sb	0	0	0	19.69	0	0	0	18.66
	Sb	0	0	0	19.74	0	0	0	18.71
	W	33.00	81.00	120.00	37.00	6.06	24.19	24.19	26.20
	Cl	0	0	0	0	0	0	0	0
(7) W-Sb									
	W	9.00	36.00	36.00	16.50	0	60.00	0	12.69
	Sb	0	0	0	24.76	0	0	0	23.47
(8) C-C-Sb-W-Cl									
	C	4.00	18.00	71.43	50.71	6.00	24.00	24.00	47.45
	C	0	0	0	52.58	5.00	20.00	20.00	50.37
	Sb	0	0	0	18.06	0	0	0	17.12
	W	6.00	24.00	24.00	32.00	5.43	28.57	28.57	30.23
	Cl	0	0	0	0	0	0	0	0
(9) C-C-Sb-W(Cl)									
	C	5.08	18.50	29.04	35.42	5.58	22.90	27.65	32.78
	C	39.84	30.82	35.78	49.69	6.00	21.58	21.58	31.77
	Sb	0	0	0	24.51	0	0	0	23.23
	W	9.00	23.25	23.25	30.88	5.43	28.57	28.57	28.17
(10) C-Sb-Rt-Rt-Rt									
	C	49.85	33.89	69.67	73.89	0	0	0	48.03
	Sb	0	0	0	24.84	0	0	0	23.54
	Rt Rt Rt	0	0	0	82.00	0	0	0	82.00
(11) C-Rt-Rt-Rt-Sb									
	C	4.00	12.50	37.00	45.00	0	0	0	39.79
	Rt Rt Rt	0	0	0	83.67	0	0	0	83.67
	Sb	0	0	0	19.80	0	0	0	18.77
(12) Sb-W-Rt-Rt-Rt-Rt									
	Sb	0	0	0	16.22	0	0	0	15.37
	W	36.55	14.04	14.85	22.91	8.00	32.00	32.00	12.27
	Rt Rt Rt Rt	0	0	0	147.93	0	0	0	147.93
(13) C-C-C									
	C	28.58	35.27	54.55	47.48	4.49	14.20	31.54	34.04
(14) Sb-Sb-Sb									
	Sb	0	0	12.00	23.75	0	0	0	22.51

Appendix Table 4. — Crop Costs Classified as Variable Costs per Acre for Growing and Harvesting Illinois Crops (adjusted to 1958 price level)

Item	Corn[a]	Soybeans[a]	Wheat[a]	Oats[a]	Establish and maintain clover seeding[a]	Redtop seed[b]
Power, repairs and fuel	$4.20	$3.65	$2.00	....	$ .25	$ .67
Machinery, repairs and fuel	4.00	3.15	2.70	....	.20	.73
Seed	2.00	3.00	3.30	....	6.00	2.10
Other crop expenses	1.50	.60	1.00	....	.10	.10
Total excluding depreciation	11.70	10.40	9.00	....	6.55	3.60

[a] Hinton, R. A., "Farm Management Manual," Dept. Agr. Econ., Univ. Ill., Mimeo AE-3349, Jan. 1959. Table 2, p. 3.

[b] Data arrived at in a conference with V. W. Davis, Agricultural Economist, ERS, USDA.

Appendix Table 5. — Model for the Flanagan Soils Farm Unit, 1954

	C_i		P_0	C_j[a] −12.5847 C-Sb-W-Cl P_1	−10.1455 C-Sb-O-Cl P_2	−17.4777 C-Sb-W(Cl) P_3	− 9.7534 C-Sb-O(Cl) P_4
P_{16}	0	Land	152	1	1	1	1
P_{17}	0	Labor W_1[b]	160	.030000	.154000	.040000	.205333
P_{18}	0	W_2	518	2.04050	2.149000	2.72067	2.71533
P_{19}	0	W_3	481	1.35700	1.30725	1.50933	1.44300
P_{20}	0	W_4	168	.720000	.567000	.960000	.756000
P_{21}	+1.34	Corn	0	−17.9650	−17.7300	−31.1000	−25.1433
P_{22}	+2.34	Soybeans	0	−8.0500	−7.4750	−9.8833	−9.9000
P_{23}	+1.93	Wheat	0	−10.4950	0	−11.8533	0
P_{24}	+ .61	Oats	0	0	−15.6900	0	−17.9800

	C_j[a] − 9.2957 C-Sb P_5	−12.1821 C-Sb-O P_6	−18.6588 C-Sb-W P_7	−13.1191 C-O-Cl P_8	−9.8231 C-C-Sb-O-Cl P_9	−16.4039 C-C-Sb-W (Cl) P_{10}
P_{16}	1	1	1	1	1	1
P_{17}	0	.165333	0	.205333	.123200	.030000
P_{18}	3.63900	2.71533	2.72067	1.80333	2.53760	3.06350
P_{19}	1.31200	1.44300	1.50933	.956333	1.09860	1.19800
P_{20}	1.13400	.756000	.960000	.638000	.836400	1.19800
P_{21}	−33.4100	−34.0000	−32.5333	−31.6100	−30.3140	−37.5225
P_{22}	−11.4200	−9.4900	−10.3100	0	−5.2320	−7.6250
P_{23}	0	0	−12.8166	0	0	−7.6675
P_{24}	0	−20.7867	0	−13.0000	−8.0100	0

	C_j[a] −11.7962 C-C-Sb-O(Cl) P_{11}	−14.6782 C-C-Sb P_{12}	−14.4245 C-C-O-Cl P_{13}	−9.9968 C-C-C P_{14}	−15.5562 C-Sb-C-O P_{15}	0 Land disposal P_{16}
P_{16}	1	1	1	1	1	1
P_{17}	.154000	0	.154000	0	.124000	0
P_{18}	3.05950	3.79000	2.37550	4.09200	3.05950	0
P_{19}	1.14825	.962667	.783250	.264000	1.14825	0
P_{20}	1.04550	1.39400	.95700	1.91400	1.04550	0
P_{21}	−35.8025	−54.0667	−38.2325	−59.8100	−46.5350	0
P_{22}	−5.8275	−9.9733	0	0	−6.6825	0
P_{23}	0	0	0	0	0	0
P_{24}	−13.1975	0	−18.0125	0	−9.9825	0

	C_j[a] 0 Labor W_1 disposal P_{17}	0 Labor W_2 disposal P_{18}	0 Labor W_3 disposal P_{19}	0 Labor W_4 disposal P_{20}	+1.34 Corn sales P_{21}	+2.34 Soybean sales P_{22}	+1.93 Wheat sales P_{23}	+.61 Oat sales P_{24}
P_{16}	0	0	0	0	0	0	0	0
P_{17}	1	0	0	0	0	0	0	0
P_{18}	0	1	0	0	0	0	0	0
P_{19}	0	0	1	0	0	0	0	0
P_{20}	0	0	0	1	0	0	0	0
P_{21}	0	0	0	0	1	0	0	0
P_{22}	0	0	0	0	0	1	0	0
P_{23}	0	0	0	0	0	0	1	0
P_{24}	0	0	0	0	0	0	0	1

[a] The C_js for $P_1 \ldots {}_{15}$ are negative prices equal to their per-unit variable costs (variable cost per rotation acre). The C_js for $P_{16} \ldots {}_{20} = 0$ since idle land and labor have no opportunity cost in this model. The C_js for $P_{21} \ldots {}_{24}$ are market prices less marketing costs for the products produced.

[b] Labor supplies are: W_1, March; W_2, October; W_3, April, May and June; W_4, July, August and September.

Appendix Table 6. — Model for the Flanagan Soils Farm Unit, 1958

			C_j[a]	−13.9907	−14.8789	−20.5083	−12.7072
				C-Sb-W-Cl	C-Sb-O-Cl	C-Sb-W(Cl)	C-Sb-O(Cl)
	C_i		P_0	P_1	P_2	P_3	P_4
P_{16}	0	Land	152	1	1	1	1
P_{17}	0	Labor W_1[b]	160	.0300	.1540	.0400	.205333
P_{18}	0	W_2	518	2.0405	2.1490	2.72067	2.71533
P_{19}	0	W_3	481	1.3570	1.30725	1.50933	1.4430
P_{20}	0	W_4	168	.7200	.5670	.9600	.7560
P_{21}	+1.34	Corn	0	−19.190	−22.7675	−30.3333	−25.1433
P_{22}	+2.34	Soybeans	0	−8.2375	−8.50	−10.5167	−9.900
P_{23}	+1.93	Wheat	0	−12.270	0	−15.0867	0
P_{24}	+0.61	Oats	0	0	−19.2775	0	−17.980

C_j[a]	−17.7201	−17.9553	−20.6882	−14.1685	−18.4814	−23.1476
	C-Sb	C-Sb-O	C-Sb-W	C-O-Cl	C-C-Sb-O-Cl	C-C-Sb-W (Cl)
	P_5	P_6	P_7	P_8	P_9	P_{10}
P_{16}	1	1	1	1	1	1
P_{17}	0	.165333	0	.205333	.1230	.030
P_{18}	3.6390	2.71533	2.72067	1.80333	2.53760	3.06350
P_{19}	1.3120	1.4430	1.50933	.956333	1.0986	1.1980
P_{20}	1.1340	.7560	.960	.6380	.83640	1.1980
P_{21}	−42.660	−40.00	−34.5667	−29.01	−40.5040	−45.4850
P_{22}	−15.670	−12.1567	−10.5433	0	−6.0720	−8.6750
P_{23}	0	0	−15.590	0	0	−9.59250
P_{24}	0	−25.070	0	−20.0	−14.910	0

C_j[a]	−19.1976	−24.4244	−14.6271	−15.8225	−19.5347	0
	C-C-Sb-O(Cl)	C-C-Sb	C-C-O-Cl	C-C-C	C-Sb-C-O	Land disposal
	P_{11}	P_{12}	P_{13}	P_{14}	P_{15}	P_{16}
P_{16}	1	1	1	1	1	1
P_{17}	.1540	0	.1540	0	.1240	0
P_{18}	3.05950	3.790	2.37550	4.0920	3.05950	0
P_{19}	1.14825	.962667	.783250	.2640	1.14825	0
P_{20}	1.04550	1.3940	.9570	1.9140	1.04550	0
P_{21}	−45.7650	−67.8333	−44.1375	−69.010	−47.660	0
P_{22}	−8.20250	−11.140	0	0	6.68250	0
P_{23}	0	0	0	0	0	0
P_{24}	−16.0725	0	−13.90	0	−8.33250	0

C_j[a]	0	0	0	0	+1.34	+2.34	+1.93	+0.61
	Labor W_1 disposal	Labor W_2 disposal	Labor W_3 disposal	Labor W_4 disposal	Corn sales	Soybean sales	Wheat sales	Oat sales
	P_{17}	P_{18}	P_{19}	P_{20}	P_{21}	P_{22}	P_{23}	P_{24}
P_{16}	0	0	0	0	0	0	0	0
P_{17}	1	0	0	0	0	0	0	0
P_{18}	0	1	0	0	0	0	0	0
P_{19}	0	0	1	0	0	0	0	0
P_{20}	0	0	0	1	0	0	0	0
P_{21}	0	0	0	0	1	0	0	0
P_{22}	0	0	0	0	0	1	0	0
P_{23}	0	0	0	0	0	0	1	0
P_{24}	0	0	0	0	0	0	0	1

[a] The C_js for $P_1 \ldots {}_{15}$ are negative prices equal to their per-unit variable costs (variable costs per rotation acre). The C_js for $P_{16} \ldots {}_{20} = 0$ since idle land and labor have no opportunity cost in this model. The C_js for $P_{21} \ldots {}_{24}$ are market prices less marketing costs for the products produced.

[b] Labor supplies are: W_1, March; W_2, October; W_3, April, May and June; W_4, July, August and September.

Appendix Table 7. — Model for the Cisne Soils Farm Unit, 1954

	C_i		P_0	P_1	P_2	P_3	P_4
C_j[a]				−10.2063	−19.4393	−9.18010	−12.7490
				C-Sb-W-Cl	C-Sb-W(Cl)	C-Sb	C-Sb-W
P_{15}	0	Land	123	1	1	1	1
P_{16}	0	Labor W_1[b]	154	.058000	.077333	.056000	.037333
P_{17}	0	Labor W_2	511	.185200	2.31933	3.05000	2.31933
P_{18}	0	Labor W_3	467	1.09625	1.16167	1.03300	1.16167
P_{19}	0	Labor W_4	161	.875750	1.16767	1.24000	1.16767
P_{20}	+1.32	Corn	0	−10.1712	−19.5123	−20.7700	−12.8420
P_{21}	+2.33	Soybeans	0	−6.52998	−8.65287	−11.5802	−8.00600
P_{22}	+1.92	Wheat	0	−5.54553	−6.90513	0	−11.0608
P_{23}	+ .419	Redtop seed	0	0	0	0	0

	P_5	P_6	P_7	P_8	P_9	P_{10}
C_j[a]	−9.17646	−8.92632	−8.54811	−10.16507	−11.5773	−4.82370
	C-Sb-Sb-W (Cl)	C-Sb-Sb-W-Cl	W-Sb	C-C-Sb-W-Cl	C-C-Sb-W (Cl)	C-Sb-Rt-Rt-Rt
P_{15}	1	1	1	1	1	1
P_{16}	.069500	.055600	.023000	.059600	.074500	.172400
P_{17}	2.40650	2.01520	1.76300	2.16800	2.59750	1.22000
P_{18}	1.23925	1.17140	1.44550	.995800	1.01975	1.61320
P_{19}	.967750	.774200	.695500	1.12300	1.40375	.50100
P_{20}	−7.30560	−8.26482	0	−18.9890	−18.0636	−6.97778
P_{21}	−8.31843	−7.48588	−12.1788	−3.75138	−5.94045	−4.60740
P_{22}	−5.62078	−5.62000	−7.30000	−6.32000	−7.09375	0
P_{23}	0	0	0	0	0	−44.2801

	P_{11}	P_{12}	P_{13}	P_{14}	P_{15}	P_{16}
C_j[a]	−4.03105	−4.98330	−9.20099	−7.01698	0	0
	C-Rt-Rt-Rt-Sb	Sb-W-Rt-Rt-Rt-Rt	C-C-C	Sb-Sb-Sb	Land disposal	Labor W_1 disposal
P_{15}	1	1	1	1	1	0
P_{16}	.172400	.174333	.066000	.046000	0	1
P_{17}	1.22000	.587667	3.43200	2.66800	0	0
P_{18}	1.61320	1.81517	.594000	1.47200	0	0
P_{19}	.50100	.273500	2.11200	.368000	0	0
P_{20}	−3.80000	0	−37.4802	0	0	0
P_{21}	−3.8028	−2.57798	0	−21.5907	0	0
P_{22}	0	−2.61818	0	0	0	0
P_{23}	−49.7843	−68.4038	0	0	0	0

	P_{17}	P_{18}	P_{19}	P_{20}	P_{21}	P_{22}	P_{23}
C_j[a]	0	0	0	+1.32	+2.33	+1.92	+.419
	Labor W_2 disposal	Labor W_3 disposal	Labor W_4 disposal	Corn sales	Soybean sales	Wheat sales	Redtop sales
P_{15}	0	0	0	0	0	0	0
P_{16}	0	0	0	0	0	0	0
P_{17}	1	0	0	0	0	0	0
P_{18}	0	1	0	0	0	0	0
P_{19}	0	0	1	0	0	0	0
P_{20}	0	0	0	1	0	0	0
P_{21}	0	0	0	0	1	0	0
P_{22}	0	0	0	0	0	1	0
P_{23}	0	0	0	0	0	0	1

[a] The C_js for $P_1 \ldots {}_{14}$ are negative prices equal to their per-unit variable costs (variable costs per rotation acre). The C_js for $P_{15} \ldots {}_{19} = 0$ since idle land and labor have no opportunity cost in this model. The C_js for $P_{20} \ldots {}_{22}$ are market prices less marketing costs for the products produced.

[b] Labor supplies are: W_1, March; W_2, October; W_3, April, May and June; W_4, July, August and September.

Appendix Table 8. — Model for the Cisne Soils Farm Unit, 1958

			C_j[a]	−11.8743	−15.4782	−9.73515	−13.3268
				C-Sb-W-Cl	C-Sb-W (Cl)	C-Sb	C-Sb-W
	C_i		P_0	P_1	P_2	P_3	P_4
P_{15}	0	Land	123	1	1	1	1
P_{16}	0	Labor W_1[b]	154	.058000	.077333	.056000	.037333
P_{17}	0	W_2	511	.185200	2.31933	3.05000	2.31933
P_{18}	0	W_3	467	1.09625	1.16167	1.03300	1.16167
P_{19}	0	W_4	161	.875750	1.16767	1.24000	1.16767
P_{20}	+1.32	Corn	0	−11.0337	−14.0323	−21.4512	−14.1020
P_{21}	+2.33	Soybeans	0	−6.59248	−7.79787	−11.7752	−8.02101
P_{22}	+1.92	Wheat	0	−6.92053	−8.53847	0	−10.3208
P_{23}	+ .419	Redtop seed	0	0	0	0	0

C_j[a]	−13.6658	−13.0454	−9.19811	−10.3971	−14.1467	−8.54042
	C-Sb-Sb-W (Cl)	C-Sb-Sb-W-Cl	W-Sb	C-C-Sb-W-Cl	C-C-Sb-W (Cl)	C-Sb-Rt-Rt-Rt
	P_5	P_6	P_7	P_8	P_9	P_{10}
P_{15}	1	1	1	1	1	1
P_{16}	.069500	.055600	.023000	.059600	.074500	.172400
P_{17}	2.40650	2.01520	1.76300	2.16800	2.59750	1.22000
P_{18}	1.23925	1.17140	1.44550	.995800	1.01975	1.61320
P_{19}	.967750	.774200	.695500	1.12300	1.40375	.501000
P_{20}	−7.95598	−10.9448	0	−20.6590	−21.2761	−14.7778
P_{21}	−8.56843	−7.88588	−12.3788	−3.61138	−6.12795	−4.96744
P_{22}	−8.37078	−7.40000	−8.25000	−6.40000	−7.71875	0
P_{23}	0	0	0	0	0	−49.2000

C_j[a]	−6.40738	−5.44102	−16.3668	−8.19898	0	0
	C-Rt-Rt-Rt-Sb	Sb-W-Rt-Rt-Rt-Rt	C-C-C	Sb-Sb-Sb	Land disposal	Labor W_1 disposal
	P_{11}	P_{12}	P_{13}	P_{14}	P_{15}	P_{16}
P_{15}	1	1	1	1	1	0
P_{16}	.172400	.174333	.066000	.046000	0	1
P_{17}	1.22000	.587667	3.43200	2.66800	0	0
P_{18}	1.61320	1.81517	.594000	1.47200	0	0
P_{19}	.501000	.273500	2.11200	.368000	0	0
P_{20}	−9.00000	0	−47.4802	0	0	0
P_{21}	−3.96028	−2.70298	0	−23.7507	0	0
P_{22}	0	−3.81818	0	0	0	0
P_{23}	−50.2031	−73.9688	0	0	0	0

C_j[a]	0	0	0	+1.32	+2.33	+1.92	+ .419
	Labor W_2 disposal	Labor W_3 disposal	Labor W_4 disposal	Corn sales	Soybean sales	Wheat sales	Redtop seed sales
	P_{17}	P_{18}	P_{19}	P_{20}	P_{21}	P_{22}	P_{23}
P_{15}	0	0	0	0	0	0	0
P_{16}	0	0	0	0	0	0	0
P_{17}	1	0	0	0	0	0	0
P_{18}	0	1	0	0	0	0	0
P_{19}	0	0	1	0	0	0	0
P_{20}	0	0	0	1	0	0	0
P_{21}	0	0	0	0	1	0	0
P_{22}	0	0	0	0	0	1	0
P_{23}	0	0	0	0	0	0	1

[a] The C_js for $P_1 \ldots {}_{14}$ are negative prices equal to their per-unit variable costs (variable costs per rotation acre). The C_js for $P_{15} \ldots {}_{19} = 0$ since idle land and labor have no opportunity cost in this model. The C_js for $P_{20} \ldots {}_{23}$ are market prices less marketing costs for the products produced.

[b] Labor supplies are: W_1, March; W_2, October; W_3, April, May and June; W_4, July, August and September.

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